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# Hidden sector renormalization of MSSM scalar masses

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Abstract: Running of gauge couplings in the MSSM from a unified value at high energies leads to a successful prediction of the weak mixing angle. Supersymmetric models at the TeV scale may contain further hints of high scale physics, such as the pattern of superpartner masses when evolved from the TeV scale to a high scale using the renormalization group. This running is traditionally assumed to be independent of effects in the hidden sector. In this paper we re-examine this assumption, and conclude that the predictions for scalar masses may depend sensitively on the details of the mechanism of supersymmetry breaking. We identify mass relations that persist even when such effects are taken into account.

Keywords: Supersymmetry Breaking, Supersymmetry Phenomenology, Supersymmetric Standard Model, GUT.

## Contents





## 1. Introduction

Weak scale supersymmetry is an attractive scenario for physics at the TeV scale. The minimal supersymmetric standard model (MSSM) with R-parity stabilizes the Higgs mass against radiative corrections, predicts weakly interacting dark matter, and satisfies constraints from precision electroweak measurements which rule out many other extensions of the standard model. With the additional assumptions of dynamical supersymmetry breaking in a hidden sector and flavor universal mediation of this breaking, the model also explains the enormous hierarchy between the electroweak scale and the Planck scale by dimensional transmutation while accommodating the constraints from flavor changing neutral currents (FCNCs).

But an especially compelling feature of the MSSM is its correct prediction of the weak mixing angle from gauge coupling unification [1–3]. This prediction supports the notion of a "desert" between the weak scale and the unification scale at $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV, and raises the hope that TeV scale measurements may allow indirect access to high scale physics through extrapolation with the renormalization group.

Can we reliably predict other TeV scale observables from high scale models? Or, working from the bottom up, can we probe high scale physics through TeV scale measurements? Previous work has indicated that superpartner masses may be just such observables [4–13].[1] These investigations imply that superpartner masses depend on the means by which SUSY breaking is communicated to the visible world, but not on the details of the hidden sector itself. If correct, this would suggest that models with simple messenger physics are

[1]See [14] for a study of whether superpartner mass relations can be tested with sufficient precision at colliders.

highly predictive. For this reason substantial effort has gone into refining [15, 16] and even automating [17 – 20] the renormalization group evolution of superpartner masses.

In this paper we demonstrate that this conventional wisdom may be wrong. While MSSM evolution of gaugino masses is reliable, we find that the renormalization group evolution of MSSM scalar masses may depend strongly on the unknown dynamics in the hidden sector. The essential point is simple. Even though the visible and hidden sectors are only coupled through higher dimensional operators, and therefore renormalizable couplings in the two sectors run independently, the higher dimensional operators themselves are renormalized by both sectors. Furthermore, the renormalization of these operators does not factor into a visible and a hidden contribution, as would naïvely seem to be the case, because of an additive contribution to scalar masses from couplings of MSSM gauginos to the hidden sector. Therefore hidden sector interactions must be taken into account when computing the renormalization of MSSM scalar masses. This may introduce dependence on new (and unknown) parameters in predictions for scalar masses. In some cases these effects are small, while in others they can even dominate over the usual one-loop MSSM running. In all cases, this dependence can be summarized in terms of a few new parameters, and model-independent predictions remain.

Before demonstrating this result in a simple toy model we summarize the main consequences of hidden sector contributions to visible sector running. For simplicity we focus on the renormalization of first and second generation scalar masses in this paper wherein the MSSM Yukawa couplings can be neglected.

We begin by considering grand unified models (and high-scale messenger sectors) wherein the operators responsible for scalar masses have unified coefficients at $M_{\rm GUT}$. Ignoring hidden sector interactions, MSSM renormalization would then give simple mass relations at the TeV scale which follow from unification at the GUT scale. We find that hidden sector renormalization can greatly modify these relations. However, one linear combination of scalar masses runs independently of hidden sector effects at one loop order in visible (MSSM) couplings and to all orders in hidden sector couplings. This allows us to predict

$$m^2_{\widetilde{Q}} - 2m^2_{\widetilde{U}} + m^2_{\widetilde{D}} - m^2_{\widetilde{L}} + m^2_{\widetilde{E}} = 0 \tag{1.1}$$

at the TeV scale. This result holds for any unified theory in which no hypercharge D-term is generated at the GUT scale.

Hidden sector running can also be important in models of gauge mediation [21 – 30]. Again, if the hidden sector couplings are strong, scalar mass predictions may be significantly modified. Nonetheless, working at one loop order in MSSM couplings and to all orders in the hidden sector, there are two linear combinations of first generation scalar masses for which hidden sector effects drop out. In addition to (1.1) we predict

$$3(m^2_{\widetilde{D}} - m^2_{\widetilde{U}}) + m^2_{\widetilde{E}} = 0 \tag{1.2}$$

at the TeV scale, in models of gauge mediation.

Hidden sector models in which strong coupling persists over a large range of scales often have large hidden sector anomalous dimensions [31 – 34]. The running from the



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hidden sector may then drive scalar masses to values which are hierarchically different from gaugino masses at a scale somewhere between the GUT and TeV scales. In the case where these anomalous dimensions cause scalar masses to become very small at the intermediate scale, they are subsequently regenerated through gaugino masses at energies below the intermediate scale. The superpartner spectrum that results in this case is that of gaugino mediation [35 – 37].

In the case where scalar masses are driven hierarchically larger than gaugino masses the model fails to stabilize the electroweak scale and requires fine-tuning. Therefore this running places new constraints on hidden sector dynamics. Theories which predict very heavy scalars [38, 39] may nevertheless be interesting in the context of anthropics and the landscape.

There are also hidden sector models with only very weak interactions. In such models the effects discussed in this paper may be negligible. Examples include the Polonyi model, and supergravity models in which supersymmetry is broken only by moduli with couplings suppressed by $M_{\rm Pl}$.

The renormalization effects we are describing may be understood in a simple toy model. Consider a hidden sector with a single chiral superfield $X$ with superpotential interaction[2]

$$\mathcal{W}_h = \frac{\lambda}{3!} X^3 \ . \tag{1.3}$$

We couple $X$ to a single generation of MSSM matter fields $\Phi_i = [Q, U, D, L, E]_i$ and SU(3)× SU(2)× U(1) gauge fields $W_n$ with the usual non-renormalizable interactions produced at the messenger scale $M$:

$$\int d^4\theta \, k_i \frac{X^\dagger X}{M^2} \Phi_i^\dagger \Phi_i + \int d^2\theta \, w \frac{X}{M} W_n W_n \tag{1.4}$$

For definiteness, we consider a grand unified model with a messenger scale $M$ at or above the GUT scale. Unification then predicts that the complex coefficient $w$ at the messenger scale is independent of the standard model gauge group. A non-renormalization theorem for $w$, which holds in holomorphic renormalization schemes,[3] implies that $w$ remains independent of gauge group at all scales. Assuming that the hidden sector field acquires a supersymmetry breaking $F$-component $\langle X \rangle|_F = F$ at the intermediate scale $M_{\rm int}$, the gaugino masses are given by the universal factor $wF/M$ times gauge couplings squared.

In the following we will not distinguish the $X$-mass scale from the scale at which supersymmetry breaks, $M_{\rm int} \sim 10^{11}$ GeV. This assumption, easily relaxed, introduces no essential changes in our analysis while substantially easing the notation.

The renormalization of the coefficients $k_i$ which determine the soft masses of the visible sector scalars $\phi_i$ is more interesting. At one loop, the $k_i$ are renormalized by visible sector

[2] This example is too simple to spontaneously break supersymmetry. We use it only to demonstrate the origin of renormalization effects from the hidden sector. A similar analysis applies in complete O'Raifeartaigh models.

[3] Throughout this paper we adopt a holomorphic scheme for all hidden sector fields as well as the MSSM gauge fields $W_n$. However we use canonically normalized MSSM matter and Higgs fields to more easily identify scalar masses.

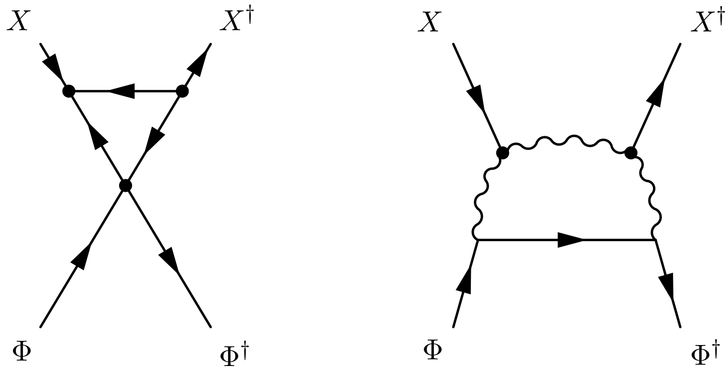


**Figure 1:** Renormalization of the operators responsible for scalar masses.

gauge interactions and the hidden sector Yukawa coupling $\lambda$ (see figure 1), and satisfy the renormalization group equations

$$\begin{aligned} \frac{d}{dt}k_i &= \frac{2\lambda^*\lambda}{16\pi^2}k_i - \frac{1}{16\pi^2}\sum_n 8\,\mathrm{C}_2^{\mathrm{n}}(R_i)\,g_n^6\;w^*w \\ &\equiv \gamma k_i - \frac{1}{16\pi^2}\sum_n 8\,\mathrm{C}_2^{\mathrm{n}}(R_i)\,g_n^6 G\ . \end{aligned} \tag{1.5}$$

Here $t = \ln(\mu/M)$, $\lambda$ is the (running) Yukawa coupling of the hidden sector, and $\mathrm{C}_2^{\mathrm{n}}(R_i)$ are group theory coefficients for the matter fields in representation $R_i$ of the $n$-th MSSM gauge group. The standard model gauge couplings $g_n$ run according to the usual MSSM RGEs. This differential equation is readily solved:

$$k_i(t) = \exp\left(-\int_t^0 dt'\,\gamma(t')\right)k_i(0) + \frac{1}{16\pi^2}\sum_n 8\,\mathrm{C}_2^{\mathrm{n}}(R_i)\int_t^0 ds\,g_n^6(s)\exp\left(-\int_t^s dt'\,\gamma(t')\right)G\ . \tag{1.6}$$

Note that with our definitions we are interested in $t < 0$, the region below the messenger scale. The exponential factor in the first term effectively rescales all scalar masses by a common factor; this can be absorbed into the messenger scale boundary value for coefficients of the operators responsible for the scalar masses, $k_i(0)$. Since only a common factor is involved, this rescaling preserves any relationships that might be present at the high scale. For example, in the case of unified boundary conditions, as would arise in SO(10), this rescaling preserves such unified boundary conditions. The second term, an additive contribution which exists even without hidden sector renormalization, splits the scalar masses. If the anomalous dimension $\gamma$ vanishes, the second term involves only standard model gauge couplings and the parameter $w$ that determines the gaugino masses. In this case the TeV-scale scalar mass differences are related to the gaugino masses. However, when the coupling $\lambda$ is non-negligible, the second term depends on hidden sector physics, and spoils predictions that follow from these relations.

Note also that the hidden sector contributions to the second term cannot be absorbed into a change of *unified* boundary conditions at the GUT scale. The example plotted in figure 2, where we have chosen a very large (positive) value for the hidden sector anomalous dimension $\gamma$, provides a good illustration. In this case, the hidden sector renormalization

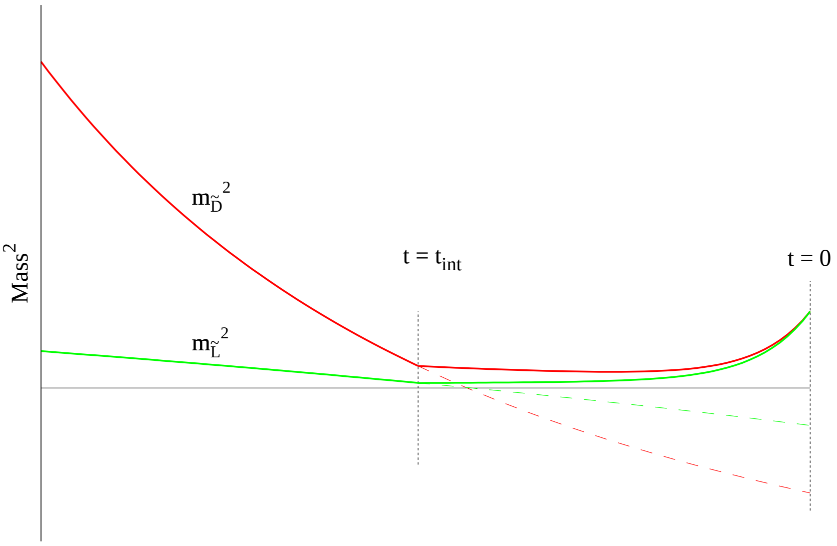


**Figure 2:** Renormalization of $\tilde{D}$ and $\tilde{L}$ scalar masses squared. The solid curves show the renormalization including hidden sector effects. The dashed curves show these masses with the same values at 1 TeV run to higher scales without hidden sector effects. Note that ignoring hidden sector effects in the running would not show unification at $t = 0$.

strongly suppresses all scalar masses at the intermediate scale. Clearly, this cannot result from MSSM running with any unified boundary condition for the $k_i(0)$ at the GUT scale.

In general, the best we can do is parameterize the masses at the intermediate scale $t_{\rm int} = \ln(M_{\rm int}/M)$ in terms of four moments

$$N_0 = \frac{|F|^2}{M^2} \exp\left(-\int_{t_{\rm int}}^0 dt'\,\gamma(t')\right) k(0) \tag{1.7}$$

$$N_n = \frac{|F|^2}{M^2}\frac{1}{16\pi^2}\int_{t_{\rm int}}^0 ds\,\exp\left(-\int_{t_{\rm int}}^s dt'\,\gamma(t')\right) g_n^6(s) G \qquad n = 1,2,3 \tag{1.8}$$

giving scalar masses

$$m_i^2(t_{\rm int}) = k_i(t_{\rm int})\frac{|F|^2}{M^2} = N_0 + \sum_{n=1}^{3} 8\,\mathrm{C}_2^{\rm n}(R_i)\;N_n\;. \tag{1.9}$$

We have assumed a unified boundary condition at the messenger scale, $k_i(0) = k(0)$. At one loop the MSSM RGE coefficients $8\ \mathrm{C}_2^{\rm n}(R_i)$ are

| | SU(3) | SU(2) | U(1) |
|---|---|---|---|
| $Q$ | 32/3 | 6 | 2/15 |
| $U$ | 32/3 | 0 | 32/15 |
| $D$ | 32/3 | 0 | 8/15 |
| $L$ | 0 | 6 | 6/5 |
| $E$ | 0 | 0 | 24/5 |

(1.10)

To obtain predictions for the physical scalar masses at 1 TeV, we should further evolve these intermediate scale masses using the MSSM RGEs. At one-loop this evolution has the same form as (1.5) (without the hidden sector effects) and may therefore be absorbed into the moments $N_n$. Inclusion of higher order MSSM running would require evolving these intermediate scale values down to the TeV scale using the higher-loop MSSM RGEs.

It is easy to check that the combination of masses in (1.1) vanishes if the hypercharge D-term vanishes at the GUT scale, as is required by the non-Abelian gauge invariance of the GUT group containing hypercharge. If we were to allow a non-vanishing hypercharge D-term, as may be the case in GUT models with unified gauge groups of higher rank [40 – 42], then the prediction (1.1) would be lost.

How significant are the contributions from the hidden sector running? For strongly coupled theories modifications of $N_0$ may be of order one or larger, whereas for weakly coupled theories they are suppressed by a loop factor times a log compared to the tree level value. However, contributions to $N_0$ can be absorbed into the unknown UV boundary condition $k_i(0)$, and are therefore of less interest. [4]

Note that even though the $N_n$ are formally suppressed by a loop factor relative to $N_0$, their contributions to scalar masses in the MSSM are numerically of similar size and split the scalar masses significantly. Unlike contributions to $N_0$, hidden sector modifications of the $N_n$ cannot in general be absorbed into UV boundary conditions without destroying relations imposed by the high scale physics. For hidden sectors which become strongly coupled at the intermediate scale, as might be expected in dynamical supersymmetry breaking models, the effects on the $N_n$ are $\mathcal{O}(1)$. Hidden sectors which remain strongly coupled for a range of scales can have even larger effects. But even weakly coupled hidden sectors may have noticeable impact. For example, if the hidden sector couplings are as weak as the MSSM couplings, then the leading non-universal contributions to scalar masses are of order $N_n^{\mathrm{MSSM}}$ times a (hidden sector) loop factor with a logarithmic enhancement from the running; this is larger by a logarithm relative to the usual two-loop MSSM contributions.

Measuring the second generation scalar masses is unlikely to give us independent information on high scale physics. This is because limits on flavor changing neutral currents already require the first and second generation scalars to be nearly degenerate. This degeneracy, once imposed at the high scale, is preserved by hidden sector renormalization, and therefore the second generation scalar masses are expected (by flavor universality) to be the same as first generation masses. The third generation is more interesting and is discussed in section 3.

## 2. General hidden sectors

The above argument is straightforward to generalize to hidden sectors with multiple chiral and vector superfields and more complex interactions. We continue to work to one-loop

[4] In the context of the MSUGRA model the hidden sector renormalization of $N_0$ is equivalent to a rescaling of $m_0$ relative to $m_{1/2}$ and $A$-terms at the GUT scale. Wave function renormalization of hidden sector fields yields a similar universal rescaling of all MSSM soft masses relative to the gravitino mass. This point has been emphasized in [32].



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order in visible sector couplings but incorporate hidden sector couplings to all orders in perturbation theory. To begin, let us introduce an efficient notation for hidden sector operators. We label all gauge invariant real superfield operators by $V_v$ and all chiral operators by $X_x$. In principle $v$ and $x$ enumerate a very large number of operators but in practice only a few of them have small enough dimension to be relevant. For convenience we will assume that the $X_x$ are normalized (with powers of the messenger scale) to have engineering dimension one, while the $V_v$ have engineering dimension two. In the example of the previous section we have $X_1 = X$ and $V_1 = X^\dagger X$. Clearly, there is an arbitrariness in the choice of basis for these operators, and $V_v$ for different $v$ may mix under renormalization. Renormalization due to hidden sector interactions needs to be taken into account from the messenger scale down to an intermediate scale $t_{\rm int}$ at which the hidden sector dynamically breaks SUSY.[5] At $t_{\rm int}$ we replace the auxiliary components of the hidden sector operators $V_v$ and $X_x$ by their expectation values

$$\langle V_v \rangle|_D = D_v \qquad\qquad \langle X_x \rangle|_F = F_x \ . \tag{2.1}$$

General hidden-visible couplings (relaxing the assumption of unified gaugino mass boundary conditions) take the form

$$\int d^4\theta\, k_{vi}\, \frac{V_v}{M^2} \Phi_i^\dagger \Phi_i + \int d^2\theta\, w_{xn} \frac{X_x}{M} W_n W_n \tag{2.2}$$

where we have suppressed all indices labeling MSSM generations. In general, the scalar mass operators may be different for the three generations, in which case the coefficients $k$ carry additional flavor indices. Continuing to suppress such indices, the superpartner masses at the intermediate scale are given by

$$\text{gaugino:} \qquad M_n = \left( \sum_x w_{xn} \frac{F_x}{M} \right) g_n^2(t_{\rm int}) \tag{2.3}$$

$$\text{scalar:} \qquad m_i^2 = \sum_v \frac{D_v}{M^2} k_{vi}(t_{\rm int}) \ . \tag{2.4}$$

The couplings $w_{xn}$ are not renormalized in perturbation theory. The $k_{vi}$ are renormalized by the diagrams in figure 3, where arbitrary hidden sector renormalizations are included through the blobs:

$$\begin{aligned} \frac{d}{dt} k_{vi} &= \gamma_{vv'}\, k_{v'i} - \frac{1}{16\pi^2} \sum_n 8\, \mathrm{C}_2^{\rm n}(R_i)\, g_n^6\, w_{xn}^* J_{vxx'}\, w_{x'n} \\ &\equiv \gamma_{vv'}\, k_{v'i} - \frac{1}{16\pi^2} \sum_n 8\, \mathrm{C}_2^{\rm n}(R_i)\, g_n^6\, G_{vn} \ . \end{aligned} \tag{2.5}$$

Repeated indices $v', x, x'$ are summed over, $\gamma_{vv'}(t)$ is the anomalous dimension matrix of the operators $V_v$ in the absence of visible sector interactions and $G_{vn}(t) \equiv w_{xn}^* J_{vxx'}(t) w_{x'n}$ are

[5] We have again ignored any difference between the scale at which SUSY breaks and the masses of the hidden sector fields. Relaxing this assumption is straightforward.


**Figure 3:** Renormalization of the operators responsible for scalar masses, including arbitrary hidden sector effects. Single solid lines represent the MSSM fields $Q$, double wavy lines represent the vector operators $V$, and double straight lines represent the chiral fields $X$.

three vectors of real functions (one vector for each standard model gauge group) determined by hidden sector interactions. Note that the blob connecting vector operators $V$ with chiral operators $X, X^\dagger$, represented by the $J$s, may include disconnected components as well as connected ones. In the absence of hidden sector interactions the anomalous dimension matrix vanishes, and $J$ simply relates the basis of free chiral fields to the vector fields $X^\dagger X$ formed as products of these chiral fields. For example the toy model of the previous section, with only one chiral field $X$ and only one vector field $X^\dagger X$, has a single blob, and $J = 1$, $G_{1n} = w^* w$ at one-loop.

In the following, we will use a matrix notation for the indices $v, v'$ labeling the vector operators. Then the RGEs become identical in form to the RGE of our simple toy model

$$\frac{d}{dt} k_i = \gamma\, k_i - \frac{1}{16\pi^2} \sum_n 8\ \mathrm{C}_2^{\mathrm{n}}(R_i)\, g_n^6\ G_n \tag{2.6}$$

and it is straightforward to generalize the solution. The only new feature is the appearance of path ordered exponentials to account for any non-commutativity of the matrices $\gamma(t)$ at different $t$

$$\begin{aligned} k_i(t) = \mathrm{P}\exp\left(-\int_t^0 dt'\,\gamma(t')\right) k_i(0) \\ + \frac{1}{16\pi^2} \sum_n 8\,\mathrm{C}_2^{\mathrm{n}}(R_i) \int_t^0 ds\, g_n^6(s) \mathrm{P}\exp\left(-\int_t^s dt'\,\gamma(t')\right) G_n(s) \end{aligned} \tag{2.7}$$

Multiplying by the (vector of) expectation values $D$ and assuming unified boundary conditions for the $k_i(0) = k$ we obtain the scalar masses at the intermediate scale

$$m_i^2(t_{\rm int}) = N_0 + \sum_{n=1}^{3} 8\ \mathrm{C}_2^{\mathrm{n}}(R_i) N_n \tag{2.8}$$

in terms of the moments

$$N_0 = \frac{D}{M^2}\ \mathrm{P}\exp\left(-\int_{t_{\rm int}}^0 dt'\,\gamma(t')\right) k \tag{2.9}$$

$$N_n = \frac{1}{16\pi^2} \frac{D}{M^2} \int_{t_{\rm int}}^0 ds\ \ g_n^6(s) \mathrm{P}\exp\left(-\int_{t_{\rm int}}^s dt'\,\gamma(t')\right) G_n(s), \quad n = 1, 2, 3 \tag{2.10}$$

## 3. The third generation

The analysis of the third generation scalar masses is similar, although complicated by the presence of a large top Yukawa coupling and possibly other interactions. The presence of this coupling necessarily connects the renormalization group flow of the top and bottom quarks with that of the Higgs scalars, and we therefore treat the two Higgs doublets along with the five matter fields of the third generation. This gives seven scalar masses in total.

The complete analysis, while straightforward, is messy, and we postpone the details for a subsequent publication [43]. However the results are easily summarized. The most predictive case applies to theories with a single, universal soft mass for all matter and Higgs scalars, all Yukawa couplings small except for the top Yukawa, and $A$-terms proportional to Yukawas (this is the case for minimal supergravity). The common soft mass may be eliminated from predictions by subtracting masses from the first generation. The presence of the large top Yukawa, which introduces a new moment when hidden sector renormalization is incorporated, requires forming linear combinations that run independently of this coupling. This leads to six predictions:

$$\begin{aligned}
2m^2_{\widetilde{Q}_3} - m^2_{\widetilde{U}_3} - m^2_{\widetilde{D}_3} - 2m^2_{\widetilde{Q}_1} + m^2_{\widetilde{U}_1} + m^2_{\widetilde{D}_1} &= 0 \quad (3.1)\\
2m^2_{\widetilde{L}_3} - m^2_{\widetilde{E}_3} - 2m^2_{\widetilde{L}_1} + m^2_{\widetilde{E}_1} &= 0 \quad (3.2)\\
2m^2_{\widetilde{Q}_3} - m^2_{\widetilde{U}_3} - 2m^2_{\widetilde{Q}_1} + m^2_{\widetilde{U}_1} &= 0 \quad (3.3)\\
m^2_{\widetilde{E}_3} - m^2_{\widetilde{E}_1} &= 0 \quad (3.4)\\
3m^2_{\widetilde{U}_3} - 3m^2_{\widetilde{D}_3} + 2m^2_{\widetilde{L}_3} - 2m^2_{\widetilde{E}_3} - 2m^2_{H} + 2m^2_{\bar{H}} - 3m^2_{\widetilde{U}_1} + 3m^2_{\widetilde{D}_1} - 2m^2_{\widetilde{L}_1} + 2m^2_{\widetilde{E}_1} &= 0 \quad (3.5)\\
-3m^2_{\widetilde{D}_3} - m^2_{\widetilde{E}_3} + 2m^2_{\bar{H}} + 3m^2_{\widetilde{D}_1} - 2m^2_{\widetilde{L}_1} + m^2_{\widetilde{E}_1} &= 0 \quad (3.6)
\end{aligned}$$

Relaxing the condition of universal soft masses, or incorporating large couplings leads to more parameters and hence fewer predictions. For example if the Higgs soft masses differ from those of the matter fields (but are the same for $H$ and $\bar{H}$) then the prediction (3.6) is lost. If the soft masses for $H$ and $\bar{H}$ differ, then (3.5) is also lost. Independently, if the $\mu$ term arises from the Giudice-Masiero [44] mechanism ($\mu H\bar{H}|_F \subset X^\dagger H\bar{H}|_D$) then both (3.5) and (3.6) are lost. The inclusion of singlet neutrino fields with a large Yukawa coupling would eliminate prediction (3.2). Finally if $\tan\beta$ is large then the predictions (3.3) and (3.4) are no longer valid. Thus, (3.1) remains as the only robust prediction. The others, (3.2)-(3.6) are still interesting as they function as indirect probes. For example, finding a violation of (3.2) would be indirect evidence for the presence of a singlet neutrino with a large Yukawa coupling.

## 4. Non-perturbative renormalization

Since we have already allowed for arbitrary functions $\gamma$ and $G$, the renormalization group equation for the scalars does include non-perturbative effects in the hidden sector. The gaugino mass operators $X_x\,[\sum_n w_{xn} W_n W_n]$ may receive non-perturbative renormalizations from the hidden sector. Any new terms which may be generated from non-perturbative hidden sector dynamics can only depend on the $W_n$ in the combinations $[\sum_n w_{xn} W_n W_n]$. Furthermore, contributions which are relevant to gaugino masses must be linear in $[\sum_n w_{xn} W_n W_n]$. It follows that if gaugino masses were unified at some scale (i.e. all $w_x$ independent of $n$), then the equality of ratios $M_1/g_1^2 = M_2/g_2^2 = M_3/g_3^2$ holds at all scales.

## 5. Gauge mediation

In models with gauge mediation supersymmetry breaking arises in the hidden sector and is communicated to the visible sector via shared gauge interactions. Usually the existence of messengers with standard model quantum numbers and supersymmetric masses $M$ is assumed. Integrating out these messengers generates the operators coupling hidden sector fields to MSSM fields (2.2) with coefficients which are then calculable (for a given choice of messengers). For our purposes we may be general and assume an arbitrary messenger sector with MSSM fields coupling to the messengers only through their gauge interactions. To leading (two-loop) order in the MSSM interactions, and with arbitrary messenger and hidden sectors this gives (see for example [9])

$$k_i(0) = \sum_n 8\ \mathrm{C}_2^{\mathrm{n}}(R_i)\, K_n \tag{5.1}$$

at the messenger scale $M$. Here we have suppressed indices $v$ labeling the hidden sector vector operators, and the functions $K_n$ parameterize the details of the messenger and hidden sectors. Running of the $k_i$ down to the intermediate scale $M_{\rm int}$ is governed by the same renormalization group equations as before, (2.5). The solution is

$$m_i^2(t_{\rm int}) = \frac{D}{M^2}\ \mathrm{P}\exp\left(-\int_{t_{\rm int}}^{0} dt'\,\gamma(t')\right) k_i(0) + \sum_{n=1}^{3} 8\,\mathrm{C}_2^{\mathrm{n}}(R_i) N_n \tag{5.2}$$

The two terms may be combined: defining the new moments $\widetilde{N}_n$

$$\widetilde{N}_n = \frac{D}{M^2}\ \mathrm{P}\exp\left(-\int_{t_{\rm int}}^{0} dt'\,\gamma(t')\right) K_n + N_n \tag{5.3}$$

we have

$$m_i^2(t_{\rm int}) = \sum_{n=1}^{3} 8\ \mathrm{C}_2^{\mathrm{n}}(R_i)\, \widetilde{N}_n \tag{5.4}$$

Thus the first generation scalar masses are given in terms of only three unknown moments, leading to two predictions independent of the hidden sector

$$m_{\widetilde{Q}}^2 - 2m_{\widetilde{U}}^2 + m_{\widetilde{D}}^2 - m_{\widetilde{L}}^2 + m_{\widetilde{E}}^2 = 0 \tag{5.5}$$

$$3(m_{\widetilde{D}}^2 - m_{\widetilde{U}}^2) + m_{\widetilde{E}}^2 = 0 \tag{5.6}$$

Since these combinations of masses are RG invariant at one-loop in MSSM couplings (and to all orders in hidden sector interactions) these predictions hold for the masses at all scales.



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Inclusion of higher order MSSM running would require evolving these intermediate scale values down to the TeV scale with the two-loop MSSM RGEs. This introduces a (weak) dependence on the unknown scale $M_{\text{int}}$ at the two-loop level.

## 6. Conclusions

We have demonstrated that interactions of the hidden sector introduce uncertainties into the renormalization of MSSM scalar masses. These new effects *cannot* be incorporated by simply rescaling the coefficients of scalar mass operators without modifying UV coupling relations.

Our results make testing unification in the pattern of scalar masses at the TeV scale significantly more challenging. Only when the hidden sector is weakly coupled can the scalar masses be evolved to the high scale without knowledge of hidden sector interactions. Without this knowledge, the mass relation (1.1) remains as the only model-independent test of unification. In the case of gauge mediation there is one further prediction, (1.2).

In a forthcoming paper we will present a detailed analysis of third generation scalar masses in the presence of interacting hidden sectors, and begin an exploration of specific classes of hidden sectors which allow predictions in addition to (1.1).

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